LECTURE # 19
Matrices

OBJECTIVES

The objectives of the lecture are to learn about:
• Matrices

QUESTIONS

Every student wonders why he or she should study matrices. There are many important questions:
Where can we use Matrices?
Typical applications?
What is a Matrix?
What are Matrix operations?
Excel Matrix Functions?

There are many applications of matrices in business and industry especially where large amounts of data are processed daily.

TYPICAL APPLICATIONS

Practical questions in modern business and economic management can be answered with the help of matrix representation in:
Econometrics
Network Analysis
Decision Networks
Optimization
Linear Programming
Analysis of data
Computer graphics

WHAT IS A MATRIX?

A Matrix is a rectangular array of numbers. The plural of matrix is matrices. Matrices are usually represented with capital letters such as Matrix A, B, C.

For example

\[
A = \begin{bmatrix}
-1 & 9 \\
-3 & 4
\end{bmatrix}
\quad
B = \begin{bmatrix}
47 & 62 & 70 & 56 \\
52 & 33 & 26 & 45
\end{bmatrix}
\quad
C = \begin{bmatrix}
1 & -4 \\
0 & 3 \\
5 & -2
\end{bmatrix}
\]

The numbers in a matrix are often arranged in a meaningful way. For example, the order for school clothing in September is illustrated in the table, as well as in the corresponding matrix.
<table>
<thead>
<tr>
<th>Size</th>
<th>Youth</th>
<th>S</th>
<th>M</th>
<th>L</th>
<th>XL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sweat Pants</td>
<td>0</td>
<td>10</td>
<td>34</td>
<td>40</td>
<td>12</td>
</tr>
<tr>
<td>Sweat Shirts</td>
<td>18</td>
<td>25</td>
<td>29</td>
<td>21</td>
<td>7</td>
</tr>
<tr>
<td>Shorts</td>
<td>19</td>
<td>13</td>
<td>48</td>
<td>36</td>
<td>9</td>
</tr>
<tr>
<td>T-shirts</td>
<td>27</td>
<td>7</td>
<td>10</td>
<td>24</td>
<td>14</td>
</tr>
</tbody>
</table>

The data in the above table can be entered in the shape of a matrix as follows:

\[
\begin{bmatrix}
0 & 10 & 34 & 40 & 12 \\
18 & 25 & 29 & 21 & 7 \\
19 & 13 & 48 & 36 & 9 \\
27 & 7 & 10 & 24 & 14
\end{bmatrix}
\]

**DIMENSION**
Dimension or Order of a Matrix = Number of Rows x Number of Columns

**Example**
Matrix T has dimensions of 2x3 or the order of matrix T is 2x3. ‘×’ is just the notation, it do not mean to multiply both of them.

\[
T = \begin{bmatrix}
6 & 2 & -1 \\
-4 & 0 & 7
\end{bmatrix}
\]

col 1 col 2 col 3

**ROW, COLUMN AND SQUARE MATRIX**
Suppose \( n = 1,2,3,4,\ldots \)
A matrix with dimensions 1xn is referred to as a row matrix
For example, matrix A to the right is a 1x4 row matrix.
A matrix with dimensions nx1 is referred to as a column matrix.
For example, matrix B in the middle is a 2x1 column matrix.
A matrix with dimensions nxn is referred to as a square matrix.
For example, matrix C is a 3x3 square matrix.

\[
A = \begin{bmatrix}
12 & 17 & 10 & 9
\end{bmatrix} \quad B = \begin{bmatrix}
-2 \\
0
\end{bmatrix} \quad C = \begin{bmatrix}
2 & -3 & 7 \\
6 & 8 & -1 \\
-5 & 0 & 4
\end{bmatrix}
\]

**IDENTITY MATRIX**
An identity matrix is a square matrix with 1's on the main diagonal from the upper left to the lower right and 0's off the main diagonal. An identity matrix is denoted as
I. Some examples of identity matrices are shown below. The subscript indicates the size of the identity matrix. For example, $I_n$, represents an identity matrix with dimensions $n \times n$.

$$
I_2 = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} \quad I_3 = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix} \quad I_4 = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

**MULTIPLICATIVE IDENTITY**

With real numbers, the number 1 is referred to as a multiplicative identity because it has the unique property that the product a real number and 1 is that real number. In other words, 1 is called a multiplicative identity because for any real number $n$, $1 \cdot n = n$ and $n \cdot 1 = n$. With matrices, the identity matrix shares the same unique property as the number 1. In other words, a $2 \times 2$ identity matrix is a multiplicative inverse because for any $2 \times 2$ matrix $A$, $I_2 \cdot A = A$ and $A \cdot I_2 = A$

**Example**

Given the $2 \times 2$ matrix, $A = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$

$$I_2 \cdot A = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} \begin{bmatrix}
2 & -1 \\
-3 & 4
\end{bmatrix} = \begin{bmatrix}
2 & -1 \\
-3 & 4
\end{bmatrix}
$$

$$A \cdot I_2 = \begin{bmatrix}
2 & -1 \\
-3 & 4
\end{bmatrix} \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} = \begin{bmatrix}
2 & -1 \\
-3 & 4
\end{bmatrix}
$$

Work

- $r1c1 = 1(2) + 0(-3) = 2$
- $r2c1 = 0(2) + 1(-3) = -3$
- $r1c2 = 2(1) + -1(0) = 2$
- $r2c1 = -3(1) + 4(0) = -3$
- $r1c2 = 1(-1) + 0(4) = -1$
- $r2c2 = 0(-1) + 1(4) = 4$
- $r1c2 = 2(0) + -1(1) = -1$
- $r2c2 = -3(0) + 4(1) = 4$

where 'r' is for row and 'c' is for column.